Name: _____

Instructor: _____

Math 10560, Practice Exam 3
April 16, 2025

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
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7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
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11.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)

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Multiple Choice _____

13. _____

14. _____

15. _____

Total _____

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Multiple Choice

1.(7 pts.) The series

$$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$$

(a) converges absolutely.

(b) diverges because the terms alternate.

(c) diverges even though $\lim_{n\to\infty} \frac{(-1)^{n+1}}{\sqrt{n}} = 0$.

(d) diverges because $\lim_{n\to\infty} \frac{(-1)^{n+1}}{\sqrt{n}} \neq 0$.

(e) does not converge absolutely but does converge conditionally.

2.(7 pts.) Use Comparison Tests to determine which **one** of the following series is divergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}\left(\frac{1}{2}\right)^n$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{n^3+100}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}+1}$

(d) $\displaystyle\sum_{n=1}^{\infty} 7\left(\frac{5}{6}\right)^n$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+8}$

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3.(7 pts.) Consider the following series

$$(I) \quad \sum_{n=1}^{\infty} \left(\frac{2n^2+7}{n^2+1}\right)^n \qquad (II) \quad \sum_{n=2}^{\infty} \frac{2^{1/n}}{n-1} \qquad (III) \quad \sum_{n=1}^{\infty} \frac{n!}{e^n}$$

Which of the following statements is true?

(a) (I) converges, (II) diverges, and (III) converges.

(b) They all converge.

(c) They all diverge.

(d) (I) diverges, (II) diverges, and (III) converges.

(e) (I) converges, (II) diverges, and (III) diverges.

4.(7 pts.) Which series below conditionally converges?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n^3}}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}e^n}{\sqrt{n}}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ (e) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}$

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5.(7 pts.) Consider the following series;

$$(I) \quad \sum_{n=1}^{\infty} \left(\frac{\sqrt{n}+1}{2n+1} \right)^n \qquad\qquad (II) \quad \sum_{n=2}^{\infty} \frac{(-1)^n 2^n \sqrt{n}}{(n-2)!}$$

Which of the following is **true**?

(a) Both of the series diverge.

(b) Both of the series converge.

(c) (I) converges and (II) diverges.

(d) (I) diverges and (II) converges.

(e) The ratio test applied to (II) is inconclusive.

6.(7 pts.) Which series below is the MacLaurin series (Taylor series centered at 0) for $\frac{x^2}{1+x}$?

(a) $\sum_{n=0}^{\infty} (-1)^n \; x^{n+2}$

(b) $\sum_{n=0}^{\infty} (-1)^n x^{2n}$

(c) $\sum_{n=0}^{\infty} \frac{x^{n+2}}{n+2}$

(d) $\sum_{n=2}^{\infty} \frac{(-1)^n x^{2n-2}}{n!}$

(e) $\sum_{n=0}^{\infty} x^{2n+2}$

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7.(7 pts.) Which series below is a power series for $\cos(\sqrt{x})$?

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n-\frac{1}{2}}}{(2n)!}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n+1)!}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n^2+1}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \sqrt{x}^{\,n}}{(2n)!}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n)!}$

8.(7 pts.) Calculate

$$\lim_{x\to 0} \frac{\sin(x^3) - x^3}{x^9}.$$

Hint: Without MacLaurin series this may be a long problem.

(a) $\frac{9}{7}$ (b) 0 (c) $-\frac{1}{6}$ (d) ∞ (e) $\frac{7}{9}$

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9.(7 pts.) Find a power series representation for the the function $f(x) = \ln(1-x^2)$. **Hint:** $\dfrac{d}{dx}\ln(1-x^2) = \dfrac{-2x}{1-x^2}$.

(a) $\displaystyle\sum_{n=0}^{\infty}(-2)^n x^{2n}$ (b) $\displaystyle\sum_{n=0}^{\infty}(-2)(2n+1)x^{2n}$ (c) $\displaystyle\sum_{n=0}^{\infty}\frac{(-2)^n x^{2n+2}}{2n+2}$

(d) $\displaystyle\sum_{n=0}^{\infty}\frac{(-2)x^{2n+2}}{2n+2}$ (e) $\displaystyle\sum_{n=0}^{\infty}\frac{(-2)^n x^{2n+1}}{2n+1}$

10.(7 pts.) Consider the function $f(x)$ defined as

$$f(x) = \sum_{n=0}^{\infty}\frac{(-1)^n x^n}{3^n n!}, \quad -\infty < x < \infty.$$

Which of the following statements is true?

(a) $\displaystyle\int_0^1 f(x)dx = \sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{3^{n+1}(n+1)!}$ (b) $\displaystyle\int_0^1 f(x)dx = \left(\sum_{n=0}^{\infty}\frac{(-1)^n}{3^n(n+1)!}\right) - 1$

(c) $\displaystyle\int_0^1 f(x)dx = \sum_{n=0}^{\infty}\frac{(-1)^n}{3^n(n+1)!}$ (d) $\displaystyle\int_0^1 f(x)dx = \left(\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{3^n n!}\right) - 1$

(e) $\displaystyle\int_0^1 f(x)dx = \sum_{n=0}^{\infty}\frac{(-1)^n}{3^n n!}$

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11.(7 pts.) What is the fourth Taylor polynomial, $T_4(x)$, for $\cos(2x)$ with center $a = \pi$?

(a) $1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4$

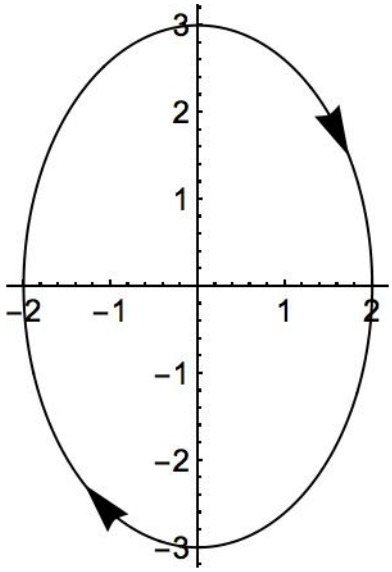
(b) $1 - \frac{1}{2!}(x-\pi)^2 + \frac{1}{4!}(x-\pi)^4$

(c) $1 - 2(x-\pi)^2 + \frac{2}{3}(x-\pi)^4$

(d) $1 + 4(x-\pi)^2 + 16(x-\pi)^4$

(e) $1 - 4(x-\pi)^2 + 16(x-\pi)^4$

12.(7 pts.)



The graph of the parametric curve shown above is the graph of which of the following parametric equations?

(a) $x(t) = 3\cos(t), \quad y(t) = 2\sin(t), \quad 0 \le t \le 2\pi.$

(b) $x(t) = 2\cos(t), \quad y(t) = 3\sin(t), \quad 0 \le t \le 2\pi.$

(c) $x(t) = 2\sin(t), \quad y(t) = 3\cos(t), \quad 0 \le t \le 2\pi.$

(d) $x(t) = \frac{3}{2}\sin(t), \quad y(t) = \cos(t), \quad 0 \le t \le 2\pi.$

(e) $x(t) = 3\sin(t), \quad y(t) = 2\cos(t), \quad 0 \le t \le 2\pi.$

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Partial Credit

You must show your work on the partial credit problems to receive credit!

13.(12 pts.) Find the radius of convergence and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} (x-3)^n$$

Remark: The correct answer with no justification is worth 2 points.

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14. (12 pts.)

(a) Show that

$$\sum_{n=0}^{\infty}(-1)^n x^{2n} = \frac{1}{1+x^2}$$

provided that $|x| < 1$.

(b) Find

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)(\sqrt{3})^{2n+1}}.$$

(**Hint:** First use term-by-term integration on the series in part (a).)

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15. (6 pts.) Please circle "TRUE" if you think the statement is true, and circle "FALSE" if you think the statement is False.

(a)(1 pt. No Partial credit) The series $\sum_{n=1}^{\infty} \frac{2^n}{3^n}$ converges by the nth root test.

TRUE FALSE

(b)(1 pt. No Partial credit) The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

TRUE FALSE

(c))(1 pt. No Partial credit) The first three terms of the power series $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n+3)!}$ are given by $1 + \frac{x^2}{2!} + \frac{x^4}{4!}$

TRUE FALSE

(d))(1 pt. No Partial credit) If $f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{2n}$, then $f''(0) = \frac{1}{4}$

TRUE FALSE

(e))(1 pt. No Partial credit) The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ diverges.

TRUE FALSE

(f))(1 pt. No Partial credit) $\cos(x) = \sum_{n=0}^{\infty} \frac{x^n}{(2n)!}$, for all values of x.

TRUE FALSE

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